Multipartite quantum nonlocality using functional Bell inequalities

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Abstract: We show that arbitrary functions of continuous variables can be used to generate tests of local hidden variable theories. The effect of nonideal detectors and noise is included, revealing that optimized functional inequalities are robust.

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Bell famously showed that the predictions of quantum mechanics (QM) are not always compatible with local hiddden variable theories (LHV). Surprisingly, this fundamental result, which underpins the field of quantum information, has not yet been rigorously verified. There are no measurements yet that can eliminate all LHV, either due to low detection efficiencies or lack of causal separation. Such rigorous tests are needed to fully implement some quantum information protocols, like that of Ekert which employs a Bell inequality (BI) as a test of security in a cryptographic scheme.

Recently, Cavalanti et al (CFRD) introduced a LHV inequality involving moments of observables that can have an arbitrary spectrum [**?**]. They showed a violation of LHV (quantum nonlocality) was possible for observables with continuous outcomes, like position and momentum, without a discretisation of the outcome domain. The CFRD approach is significant in establishing that quantum nonlocality does not rely on having discrete measurement outcomes of spin or particle number, thus allowing the use of efficient quadrature detectors. Hence, we expect quantum nonlocality, like entanglement, to be evident for a range of different obervables. Continous variable (cv) inequalities open new forms of quantum information to investigation, based on *functional* observables.

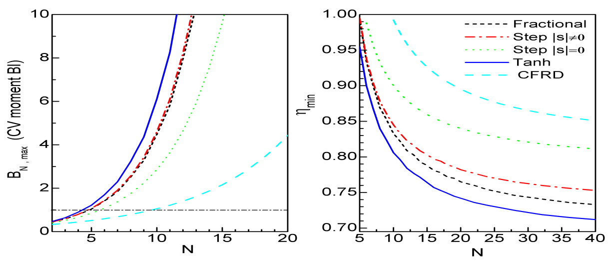

Fig. 1. Left: Maximum violations of CFRD functional cv BI with GHZ states as a function of the number of modes with ideal detection efficiency correspond to different function classes. Rigth: The minimum detection efficiency for pure state required for violation corresponding to the Left.

In this paper we formulate and develop the *functional* CFRD BI

$$|\langle\Pi_{k=1}^{N}(f_k(x_k)+ig_k(p_k))\rangle|^2 \leq \langle\Pi_{k=1}^{N}(f_k(x_k)^2+g_k(p_k)^2)\rangle, \quad (1)$$

where $f_k(x_k)$, $g_k(p_k)$ are arbitrary functions of the outcome x_k, p_k at the kth site. We show that by optimising the choice of function, one can produce a BI which is violated by the symmetric GHZ states for $N \geq 5$. The violation increases exponentially with N, while the critical efficiency η_{crit} decreases asymptotically, to 0.72, as $N \to 40$ using the optimal tanh function. This dramatically reduces both the number of modes required, and the required efficiency. The outcome is a BI that could be feasibly tested in the laboratory, since N=6 GHZ states have already been generated. More generally, this insight into functional quantum information measures could lead to new applications of quantum information.

References

1. E. G. Cavalcanti, C. J. Foster, M. D. Reid, and P. D. Drummond, Phys. Rev. Lett. **99**, 210405 (2007).